FRICTION DURING DOWNWARD FILM FLOW ALONG A VERTICAL WALL

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Results of an experimental study are shown where the shearing stresses at a vertical wall during downward film flow of a liquid were measured by the electrodiffusion method. The film thickness was measured by the shadowgraph method.

A study of the mechanism by which liquid films flow down along vertical walls is important, if one considers the wide technological application of film-flow apparatus.

The authors present here results of electrodiffusion measurements covering the average friction at a vertical surface, this method having been proposed earlier [1, 2] for one-phase streams.

The gist of the electrodiffusion method is as follows. If a direct current is passed through an electrolytic cell (anode—electrolyte film—cathode), then the following reactions take place at the cathode and at the anode respectively:

$$Fe(CN)_6^{3-} + e \rightarrow Fe(CN)_6^{4-},$$

$$Fe(CN)_6^{4-} \rightarrow Fe(CN)_6^{3-} + e.$$

When the diffusion current is maximum, then the mass flow toward the measuring electrode can be determined according to the relation

$$q = \frac{I}{F\Phi}(1-T), \tag{1}$$

with the transport number T equal to 0.001 for the parameters of our particular electrolyte solution.

Let one electrode (the cathode, for example) represent a conductive and hydrodynamically smooth area, mounted into a dielectric plate along which the electrolyte is flowing. From an analysis of diffusive transfer at the electrode one can now establish the relation between the mass flow toward the electrode surface and the friction at that surface. In our case the process at the electrode is characterized by very high values of the Prandtl diffusion number $Pr \gtrsim 1000$. As a consequence, the rather thin diffusion layer at the probe surface is always contained within the laminar sublayer, where the velocity profile is linear [3].

If x denotes the longitudinal coordinate referred to the probe and y denotes the transverse coordinate, then the equation of a diffusive boundary layer is

$$\frac{\tau_w}{\mu} y \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2}, \tag{2}$$

for steady flow, with $C(x, 0) = 0$ in the diffusion mode and $C(x, \infty) = C_\infty$. A simultaneous solution of Eqs. (2) and (1) yields the following relation between the shearing stress at the electrode and the maximum diffusion current:

$$\tau_w = \frac{1{,}85\mu I^3}{\Phi^3 D^2 C_\infty^3 l^2 b^3},$$

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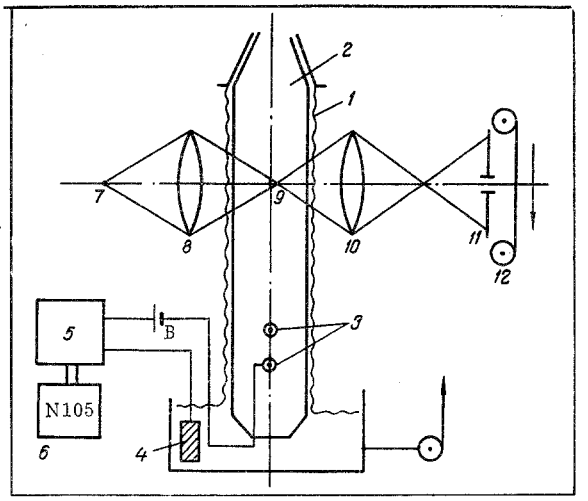

Fig. 1

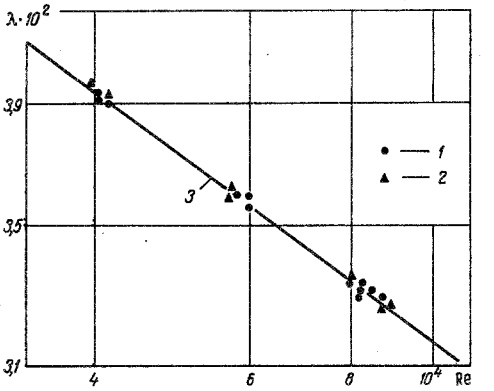

Fig. 2

Fig. 1. Schematic diagram of the test apparatus: 1) film of liquid, 2) active compartment, 3) probe, 4) anode, 5) amplifier, 6) loop oscillograph, 7) to 12) optical system.

Fig. 2. Drag coefficient λ as a function of the Reynolds number Re in an annular channel: 1) 900 mm from the entrance with L/h = 180, 2) 950 mm from the entrance with L/h = 190, 3) according to Blasius.

where l and b denote the electrode length and the electrode width respectively. In the case of a circular probe this equation becomes

$$\tau_w = \frac{3.12\mu I^3}{\Phi^3 D^2 C_\infty^3 d^5}. \qquad (3)$$

The test apparatus is shown schematically in Fig. 1. A film of liquid (electrolyte) 1 flowed down along the surface of a vertical cylindrical tube 2 made of acrylic glass. The tube was 1000 mm long and 60 mm in diameter. Flush against the tube wall were installed measuring electrodes (cathodes) 3. The other electrode (anode) 4 with a much larger surface area was placed in the lower part of the test compartment. The cathodes were made of platinum wire 0.2 mm in diameter, fused into a capillary made of chemically resistant glass with the proper thermal expansivity. This structure was hermetically cemented into the tube and this entire test assembly was carefully polished with abrasive powder. The end face of a platinum wire served as the operating surface of the electrode. As electrolyte we used a 0.01 N solution of a mixture of potassium ferrocyanide and potassium ferricyanide in distilled water with a 0.5 N solution of NaOH. All necessary physical constants were either taken from published sources [4] or measured at a 25° temperature, held in our tests within 0.2°C. On account of the small difference between the dissociation potentials of oxygen and ferrocyanide, all tests were performed in an atmosphere of nitrogen – an inert gas – and the active compartment was covered with a protective jacket.

A constant-level supply tank ensured a constant flow rate of liquid throughout the test time. Breakdown of the film, especially at low values of the Reynolds number, was prevented by the careful surface polish and a concentric alignment of the film blower cone and the entire test compartment.

A signal from the electrodiffusion probe was transmitted to a special-purpose dc amplifier 5. For reading the average current, the amplifier signal was smoothed by an electric filter network; the rms current pulsations were measured at the same time. For reading the instantaneous values of the diffusion current, the signal was recorded on a loop oscillograph 6.

The performance of probes was checked by comparing the friction data obtained by the electrochemical method with well known formulas for annular channels. As a result of these control tests, we determined the true active surface area of each probe-electrode. The protective jacket in our tests served as the outer channel wall. The radius of the inner tube and the radius of the outer tube were in a ratio 0.86, while the ratio of channel length to gap width was 200. Our measurements of the drag coefficient are compared in Fig. 2 with the well known Blasius formula $\lambda = 0.316\,\mathrm{Re}^{-1/4}$ for turbulent flow. All test points plotted here refer to the true active surface of the electrodes. The stability of probe readings depended largely on the purity of the active surface and was achieved by electrochemical activation in a pure 1 N solution of NaOH [5].

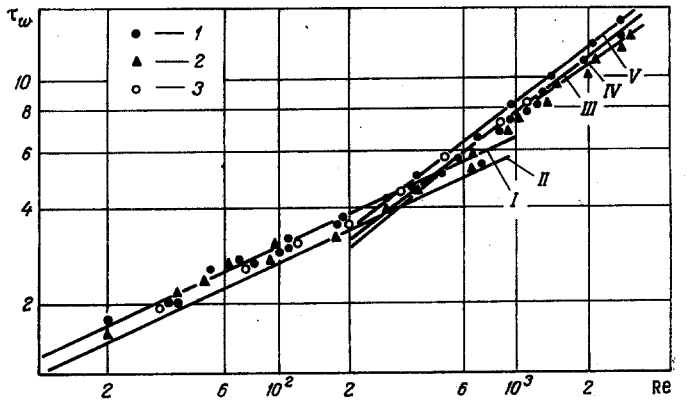

Fig. 3

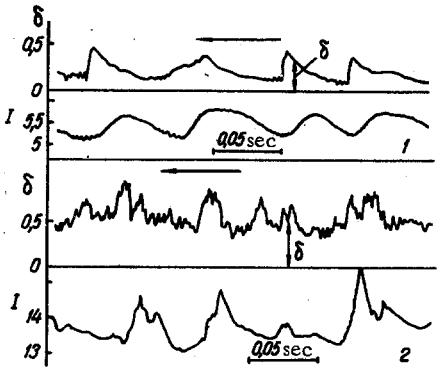

Fig. 4

Fig. 3. Shearing stress τ_W as a function of the Reynolds number. Based on experiments: 1) and 2) by the electrodiffusion method, 3) by δ_{av} measurements, 4) according to [13]. Based on calculations: I) according to [14], II) according to [15], III) according to formula (7), IV) according to [10].

Fig. 4. Instantaneous film thickness, δ, mm and pulsations of diffusion current I, μ A: 1) Re = 70; 2) Re = 890.

The instantaneous film thickness was measured by the shadowgraph method shown schematically in Fig. 1. A light beam from source 7 was focused through lens 8 on the liquid film at point 9. The diameter of the light spot at this point was always larger than the maximum possible film thickness. The shadow of the film profile was projected through objective 10, shutter 11, and a mirror system on the photographic strip of a loop oscillograph 12. The contrast of the optical system was less than 1 mm, which made it possible to separate out the examined flow region. The necessary magnification was obtained by adjusting the distance between objective 10 and shutter 11. The average film thickness was measured by tracing the oscillograms with a planimeter. In the evaluation of test data for laminar flow, the average film thickness δ_{av} was found to have been stabilized already at a distance equal to approximately ten times the film thickness away from the site of film formation. This conclusion was based on the reports by S. Gasan [6] as well as by V. S. Kasimov and F. F. Zigmund [7]. According to the report by K. Feind [8] and the estimate by P. L. Kapitsa [9], the flow was assumed two-dimensional within an active zone of the size as in our tests.

A simpler approach may also be taken for analyzing the film stabilization during turbulent flow. The momentum equation will be written as follows:

$$\frac{dG}{dx} = \rho g\delta - \tau_w,$$

with the longitudinal x-coordinate measured from the point where the film begins to form, and

$$G = \rho \int_0^\delta u^2 dy, \tag{4}$$

where G denotes the momentum of the liquid. We assume that in a turbulent stream the velocity profile is described by the Prandtl equation $u/u_S = (y/\delta)^{1/7}$ and the shearing stress at the wall τ_W is described by the Blasius equation

$$\tau_w = 0.0228 u_s (\nu/u_s\delta)^{1/4}.$$

We then insert these expressions into (4) and obtain

$$\frac{d\delta}{dx} + \frac{63}{64}\frac{g\delta^3}{Q^2} - \frac{0.0284}{\mathrm{Re}^{1/4}} = 0, \tag{5}$$

where

$$Q = \int_0^\delta u dy, \quad \mathrm{Re} = Q/\nu.$$

Integration of Eq. (5) yields $x = \Gamma - P$ with

$$P = \frac{\delta_e \mathrm{Re}^{1/4}}{0.0284 \cdot 3} \left\{ \frac{1}{2} \ln \left[\frac{(\delta - \delta_e)^2}{\delta^2 + \delta\delta_e + \delta_e^2} \right] + \sqrt{3} \operatorname{arctg} \left[-\frac{2\delta + \delta_e}{\sqrt{3}\,\delta_e} \right] \right\}. \tag{6}$$

Here $\Gamma = P$ at the point where the film begins to form $\delta = \delta_b$.

An analysis based on Eq. (6) shows that stabilization occurs within the distance (100 to 200) δ_b.

For the stabilized flow mode, Eq. (5) yields an expression for the equilibrium thickness of a smooth turbulent film:

$$\delta_e = \sqrt[3]{\frac{0.0288\nu^2 \mathrm{Re}^{7/4}}{g}} \tag{7}$$

During equilibrium flow we have

$$\tau_w = \rho g \delta_e. \tag{8}$$

Values of the average shearing stress at distances 900 and 950 mm from the channel entrance are shown in Fig. 3 for values of the Reynolds number ranging from 20 to 2800. The results of these measurements are compared here with already known theoretical and empirical relations.

According to the graphs, at a low Reynolds number (for laminar-ripple flow) there is a satisfactory agreement with the Nusselt theory [14]. Similar conclusions have been arrived at in [11, 12]. The graphs indicate a transition to turbulent flow within the Re = 400-500 range.

At high Reynolds number there is a satisfactory agreement both with the Kutateladze–Styrikovich relation [10] and with formula (7). However, formula (7) is more convenient for engineering calculations. The data obtained here agree closely with the Gimbutis–Vasilyauskas formula [13] based on direct measurements of the shearing force on the wall. The blank dots in Fig. 3 represent friction values calculated from readings of average film thickness on the basis of the flow balance hypothesis, i. e., according to the formula

$$\tau_{w_1} = \rho g \delta_{av}.$$

In Fig. 4 are shown instantaneous film thicknesses and pulsations of the diffusion current (which is uniquely related to friction), both recorded simultaneously at the same point at various values of the Reynolds number. Evidently, during laminar flow (Re = 70) there is an almost complete correspondence between fluctuations of film thickness and of shearing stress at the wall. During turbulent flow (Re = 890) the fluctuations of shearing stress follow the fluctuations of film thickness only very roughly. In both flow modes there appears a definite time shift between maximum δ and maximum τ_W during low-frequency ripples.

NOTATION

q	is the mass flow rate;
Φ	is the Faraday number;
F	is the electrode surface area;
T	is the transport number;
I	is the maximum diffusion current;
$\mathrm{Pr} = \nu/D$	is the Prandtl number;
x, y	are the longitudinal and the transverse coordinates;
τ_W	is the shearing stress at the wall;
μ	is the dynamic viscosity;
D	is the diffusivity;
l	is the electrode dimension along the liquid flow;
b	is the electrode dimension perpendicular to the liquid flow;
C_∞	is the ion concentration outside the diffusive boundary layer;
C	is the ion concentration;
d	is the electrode diameter;
λ	is the drag coefficient;
G	is the momentum;
ρ	is the density;

u is the velocity;
u_s is the velocity at the film surface;
δ is the film thickness;
ν is the kinematic viscosity;
g is the acceleration of gravity;
Q is the volume trickle rate;
$Re = Q/\nu$ is the Reynolds number;
δ_e is the equilibrium film thickness;
δ_{av} is the average film thickness;
τ_{w_1} is the average shearing stress at the wall;
L is the distance from the channel entrance;
h is the gap width of the annular channel.

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